# Anthony Roper Primary School

# Policy for written calculations

*Background to the policy*

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

In this policy we recommend the route most children should be able to follow successfully and with understanding. (In our school many children are tutored and therefore have knowledge of more complex methods without the understanding of what they are completing.) However teachers may wish to introduce other methods for both the more able or less able children to investigate and explore. If children are secure with a method then they do not need to learn another one. Every teacher MUST make sure that at the beginning of teaching a topic they put several calculations up on the board and allow the children the opportunity to show the method they are using. This will then inform future teaching.

**Introduction**

The Mathematics programmes of study: key stages 1 and 2 provides a structured and systematic approach to teaching number. There is a considerable emphasis on teaching mental calculation strategies. During their time at Anthony Roper Primary School, children will be encouraged to see mathematics as both a written and spoken language. This policy shows the written methods the children will be taught as they learn to master each of these number operations. The children will be taught each new stage when they are ready and their learning will be supplemented by a substantial amount of additional material (*e.g.* mental processing, shape space and measures, word problems, probability, fractions, decimals and percentages.) The transition between the stages should not be hurried and children must be able to explain their current method.

Although the focus of this policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally is fundamental to a child becoming numerate. Mental methods will be taught systematically throughout the school and children will be given regular opportunities to develop the necessary skills. Written methods of calculation are based on mental strategies. Each of the four operations builds on the mental skills which provide the foundations for jottings and informal written methods of recording. Skills need to be taught, practised and constantly reviewed. These skills lead on to more formal written methods of calculation. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to

develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

**Reasons for using written methods**

- To aid mental calculation by writing down some of the numbers and answers involved.
- To make clear a mental procedure for the child
- To help communicate methods and solutions
- To provide a record of the work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculation.

**Aims**

The long-term aim is for children to be able to choose an efficient method of their choice; mental or written, appropriate to the given task. This they will do by asking themselves:

**Can I do this in my head? (using rounding and adjustment)**
**Could I do jottings to keep track of the calculation?**
**The size of an approximate answer (estimation)**
**Do I need to use a pencil and paper procedure?**

By the end of Year 6, children working at the expected level and above will have been taught, and should be secure with, a compact standard method for each operation.

# PROGRESSION THROUGH CALCULATIONS FOR ADDITION

## MENTAL CALCULATIONS

**Mental recall of number bonds**

6 + 4 = 10 □ + 3 = 10

25 + 75 = 100 19 + □ = 20

**Use near doubles**

6 + 7 = double 6 + 1 = 13

**Addition using partitioning and recombining**

34 + 45 = (30 + 40) + (4 + 5) = 79

**Counting on or back in repeated steps of 1, 10, 100, 1000**

86 + 57 = 143 (by counting on in tens and then in ones)

460 - 300 = 160 (by counting back in hundreds)

**Add the nearest multiple of 10, 100 and 1000 and adjust**

24 + 19 = 24 + 20 – 1 = 43

458 + 71 = 458 + 70 + 1 = 529

**Use the relationship between addition and subtraction**

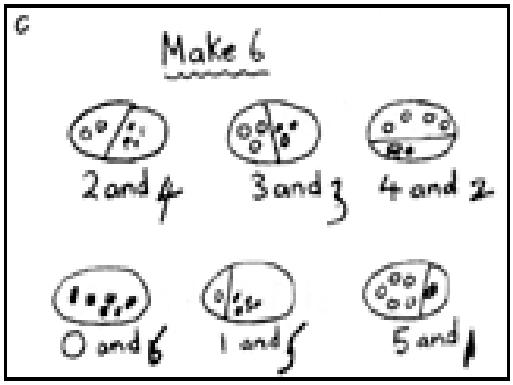
36 + 19 = 55 19 + 36 = 55

55 – 19 = 36 55 – 36 = 19

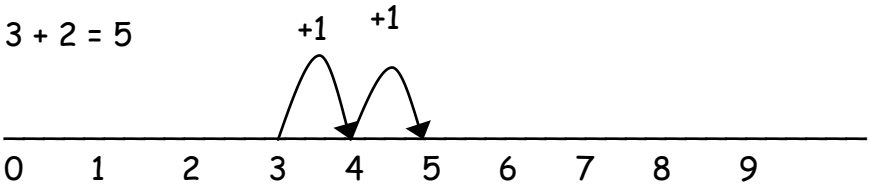
## RECEPTION and YEAR 1

In Reception, Mathematical learning will take place in the forms of whole class carpet sessions, differentiated groups and problem solving activities (where appropriate). Then there will be the opportunity for child initiated learning for children to explore number and reasoning.

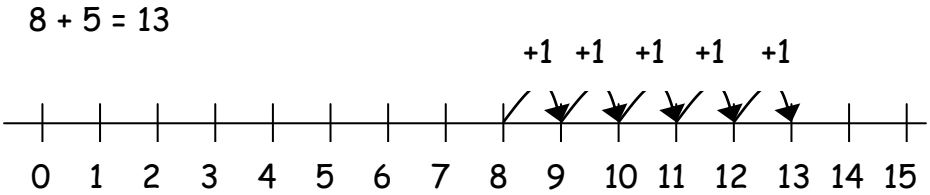
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



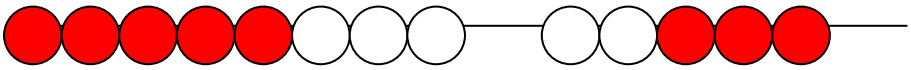

They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.




Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.




Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



Children will begin to use dienes and other place value apparatus before the end of Year 1.

## YEAR 2

Children use Dienes rods and cubes (Base 10) to partition and recombine numbers to support basic place value. E.g. 21 + 23 = 44

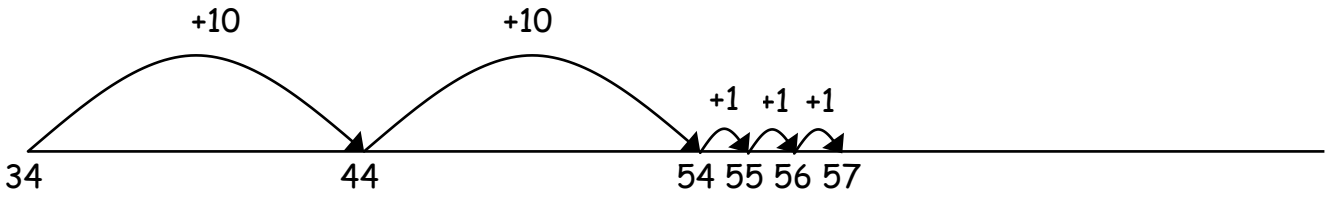
II. + II ... IIII ....

They should be able to visualise or draw the dienes to partition numbers to 100.

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

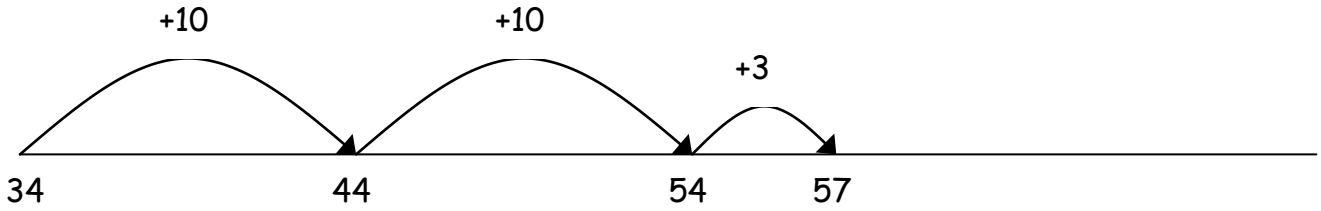
✓ First counting on in tens and ones.

34 + 23 = 57



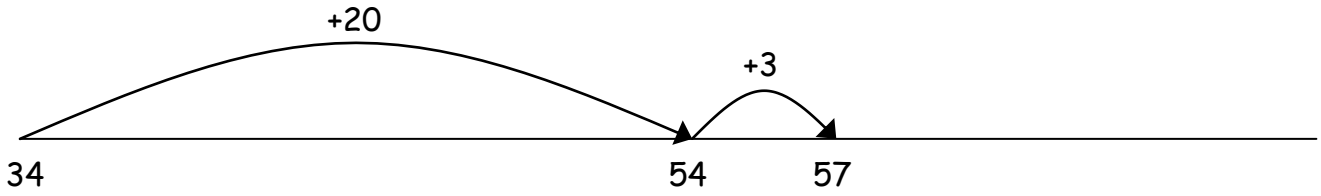

✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).

34 + 23 = 57



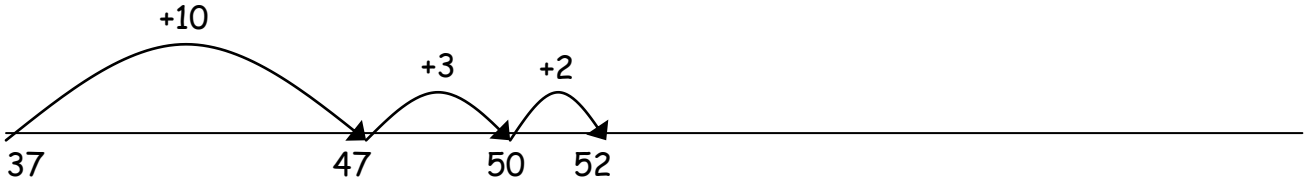

✓ Followed by adding the tens in one jump and the units in one jump.

34 + 23 = 57




✓ Bridging through ten can help children become more efficient.

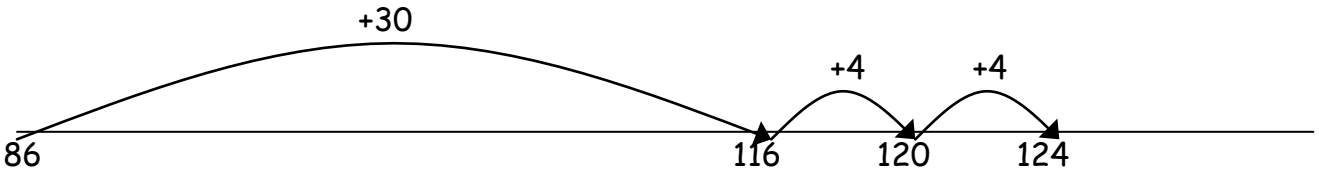
37 + 15 = 52




## YEAR 3

✓ Count on from the largest number irrespective of the order of the calculation.

INFORMAL:
38 + 86 = 124




Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies and partitioning numbers.
EXPANDED:
49 + 73

40 + 70 = 110  110 + 12 = 122
9 + 3 = 12

*By the end of Y3 all children should be familiar with and used to using the extended method even if they use a number line to support.*

## END OF YEAR 3/ YEAR 4

### VERTICAL EXPANDED

80 + 5
60 + 7
140 + 12 → 152

or

| | 80 | 5 | |
|---|---|---|---|
| | 60 | 7 | + |
| 100 | 50 | 2 | → 152 |
| | 10 | | |

**moving on to...**

```
  67
+ 24
  11 ( 7 + 4)
  80 (60 + 20)
  91
```

```
  267
+  85
   12 ( 7 + 5)
  140 (60 + 80)
  200
  352
```

*Using similar methods, children will:*
- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*
- ✓

| £ | p |
|---|---|
| 3 | . 59 |
| + 0 | . 78 |
| 0 | . 17 (8+9) |
| 1 | . 20 (70+50) |
| 3 | . 00 |
| £ 4. | 37 |

<u>similarly differentiated for lower ability</u>

£3 + 50p + 9p
+ £0 + 70p + 8p
£3 + £1.20 + 17p → £4.37

## YEAR 5

Children should continue to use partitioning method until they are confident with carrying (they can leave Year 6 still using the vertically expanded method from Year 4)

Children could extend the carrying method to numbers with at least four digits.

| | |
|---|---|
| 587 | 3587 |
| + 475 | + 675 |
| 1062 | 4262 |
| 1 1 | 1 1 1 |

*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*

## YEAR 6

(As above in Year 5- children need to be confident with one method and this can be the vertically expanded or compact method)
Children could extend the carrying method to number with any number of digits.

| | | |
|---|---|---|
| 7648 | 6584 | 42 |
| + 1486 | + 5848 | 6432 |
| 9134 | 12432 | 786 |
| 1 1 1 | 1 1 1 | 3 |
| | | + 4681 |
| | | 11944 |
| | | 1 2 1 |

Children should be encouraged to look for number bonds as a middle step when adding up the different columns.

*Using similar methods, children will*

- ✓ *add several numbers with different numbers of digits;*

- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.*

# PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

## MENTAL CALCULATIONS

**Mental recall of addition and subtraction facts**

10 – 6 = 4 17 - □ = 11
20 - 17 = 3 10 - □ = 2

**Find a small difference by counting up**

82 – 79 = 3

**Counting on or back in repeated steps of 1, 10, 100, 1000**

86 - 52 = 34 (by counting back in tens and then in ones)
460 - 300 = 160 (by counting back in hundreds)

**Subtract the nearest multiple of 10, 100 and 1000 and adjust**

24 - 19 = 24 - 20 + 1 = 5
458 - 71 = 458 - 70 - 1 = 387

**Use the relationship between addition and subtraction**

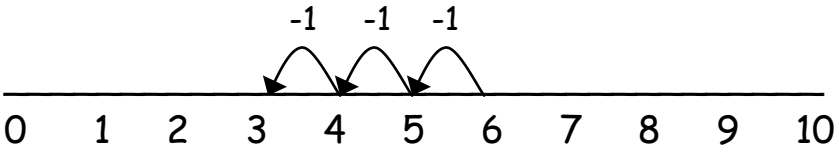
36 + 19 = 55 19 + 36 = 55
55 – 19 = 36 55 – 36 = 19

## RECEPTION and YEAR 1

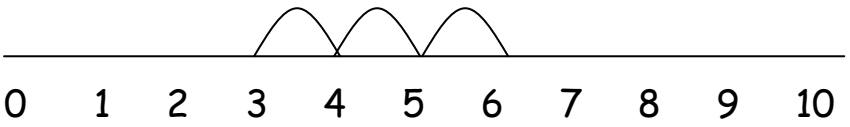
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

They use numberlines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

6 – 3 = 3

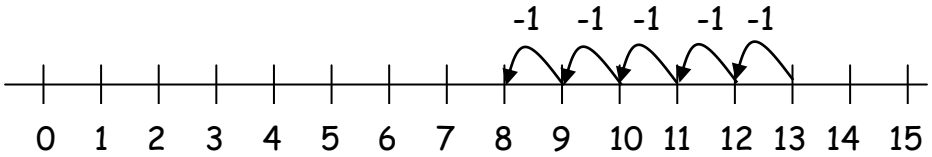



The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



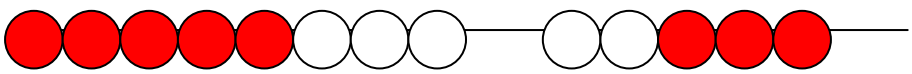

Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

13 – 5 = 8




Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

13 – 5 = 8



**YEAR 2**

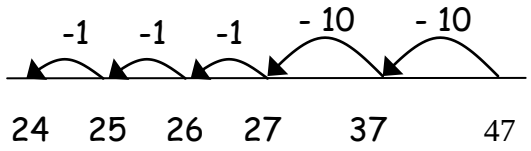
Children will use dienes to partition physically, leading to changing a ten for ones when it goes through the ten boundary.

Children will begin to use empty number lines to support calculations.

**Counting back**

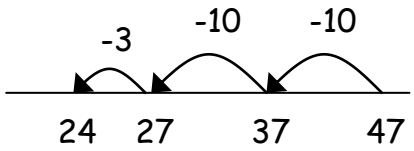
✓ First counting back in tens and ones.

47 – 23 = 24



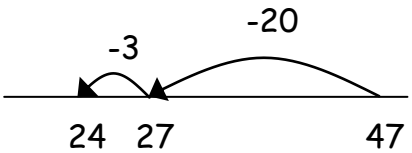

✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).

47 – 23 = 24



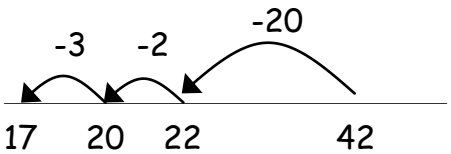

✓ Subtracting the tens in one jump and the units in one jump.

47 – 23 = 24




✓ Bridging through ten can help children become more efficient.

42 – 25 = 17

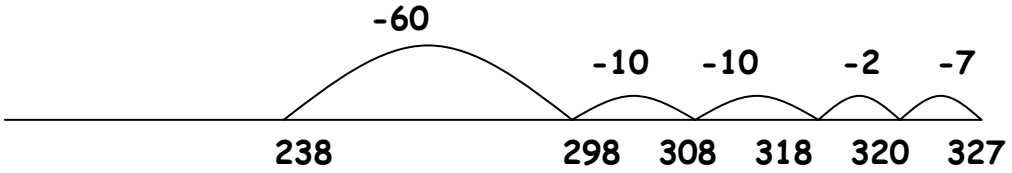


## YEAR 3

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Children will continue to use empty number lines with increasingly large numbers using the counting back method. Empty number lines will continue to be horizontal but work with counting back in known chunks.

327 – 89




Differentiation at this stage will be through the size of steps children take away. Some children will be able to combine steps such as 80, others will need to use smaller steps such as -10 -10 - 60.
This could be used to check the inverse, by counting back on.

## END OF YEAR 3/ YEAR 4

Children will now develop partitioning in a vertical method, to use decomposition.

| 327 – 89 | 200 | 110 | |
|---|---|---|---|
| | ~~300~~ | ~~20~~ | $^{1}7$ |
| | - 80 | | 9 |
| | 200 | 30 | 8 |

*Children should:*

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;*
- ✓ *subtract different numbers with decimal points.*

£25.13 - £19.18

| | 10 | 14 | 1.0 | 1 | |
|---|---|---|---|---|---|
| | ~~20~~ | ~~5~~ | ~~0.1~~ | 0.03 | |
| - | 10 | 9 | 0.1 | 0.08 | |
| | 0 | 5 | 0.9 | 0.05 | = £5.95 |

## YEAR 5

Use decomposition method until children are ready to 'exchange'.

$$\begin{array}{rr} & 8 \\ & 7\,\not{9}^{\backslash}.{}^{1}0\,7 \\ - & \underline{4\,5\,.\,3\,2} \\ & 3\,3\,.\,7\,5 \end{array}$$

## YEAR 6

Developing and refining the adding on method as outlined in Y5.

*Children should:*

- ✓ *be able to subtract numbers with different numbers of digits by lining up the columns correctly;*
- ✓ *be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other.*

# PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

**Doubling and halving**
Applying the knowledge of doubles and halves to known facts.
e.g. 8 x 4 is double 4 x 4

**Using multiplication facts**

***Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.***

| | |
|---|---|
| Year 2 | 2 times table |
| | 5 times table |
| | 10 times table |
| Year 3 | 2 times table |
| | 3 times table |
| | 4 times table |
| | 5 times table |
| | 8 times table |
| | 10 times table |
| Year 4 | Derive and recall all multiplication facts up to 12 x 12 |
| Years 5 & 6 | Derive and recall quickly all multiplication facts up to 12 x 12. |

**Using and applying division facts**
Children should be able to utilise their tables knowledge to derive other facts.
e.g. If I know 3 x 7 = 21, what else do I know?
30 x 7 = 210, 300 x 7 = 2100, 3000 x 7 = 21 000, 0.3 x 7 = 2.1 etc

**Use closely related facts already known**
13 x 11 = (13 x 10) + (13 x 1)
= 130 + 13

= 143

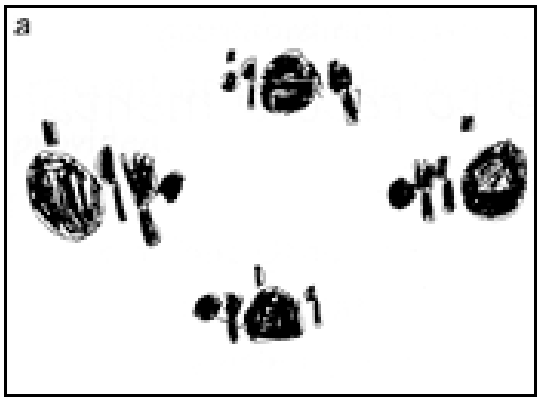
**Multiplying by 10 or 100**
Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.
Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

**Partitioning**
23 x 4 = (20 x 4) + (3 x 4)
= 80 + 12
= 102

**Use of factors**
8 x 12 = 8 x 4 x 3

## RECEPTION and YEAR 1

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.



## YEAR 2

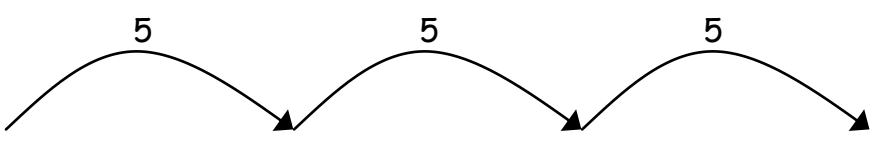
Children will develop their understanding of multiplication and use drawings to support calculations:

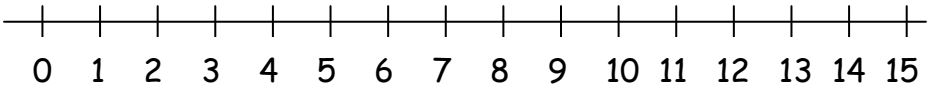
- ✓ **Repeated addition**

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

Repeated addition can be shown easily on a number line:

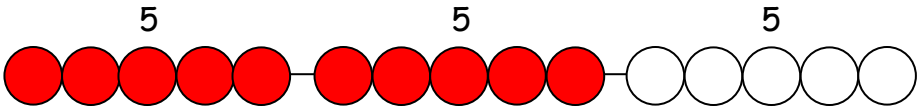
5 x 3 = 5 + 5 + 5





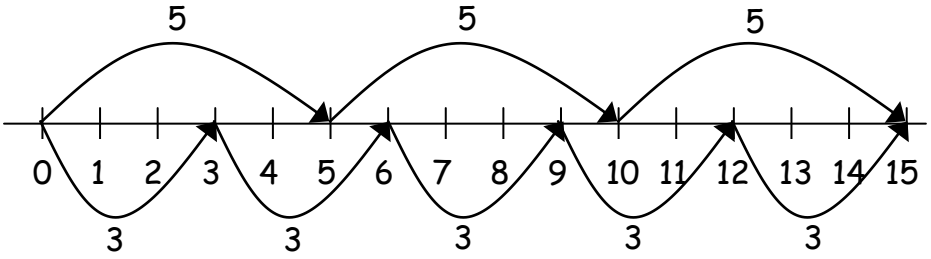

and on a bead bar:

5 x 3 = 5 + 5 + 5



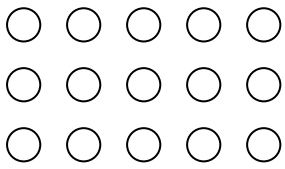

✓ **Commutativity**

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.




✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



5 x 3 = 15

3 x 5 = 15

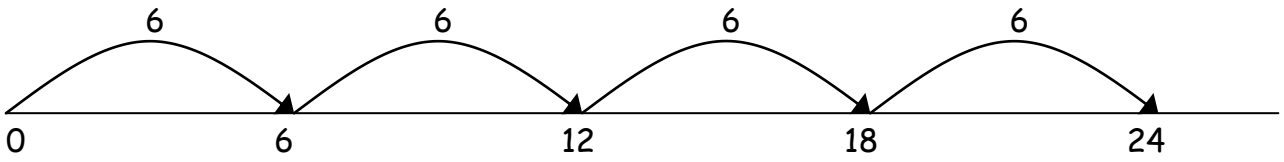
## YEAR 3

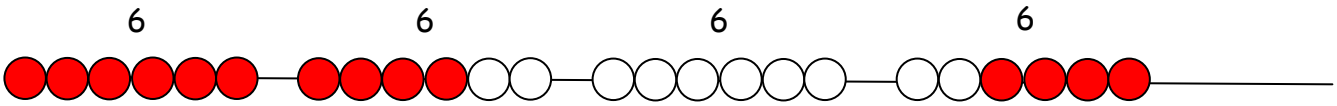
Children will continue to use:

✓ **Repeated addition**

4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

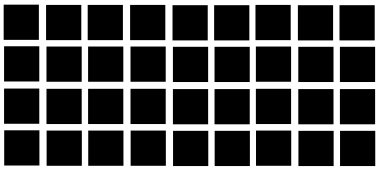
Children should use number lines or bead bars to support their understanding.




✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



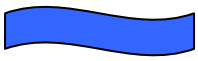
9 x 4 = 36

9 x 4 = 36

Children will also develop an understanding of

✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



5 cm

20 cm

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

□ x 5 = 20 3 x △ = 18 □ x O = 32

✓ **Partitioning using grid method**

38 x 5

| x | 30 | 8 |
|---|---|---|
| 5 | 150 | 40 |

150

+ 40

190

Or the two calculations 30 x 5 =
8 x 5 =
*Ensure that the columns are lined up.*

## YEAR 4

Children will continue to use grid method or partitioning sums.

**Grid method**

**TU x U**
(Short multiplication – multiplication by a single digit)

23 x 8

Encourage estimating first; 23 x 8 is approximately 25 x 8 = 200

| X | 20 | 3 |
|---|---|---|
| 8 | 160 | 24 |

160
+ 24
184

## YEAR 5

**Grid method**

**HTU x U**
(Short multiplication – multiplication by a single digit)

346 x 9

Children will approximate first
346 x 9 is approximately 350 x 10 = 3500

| | 300 | 40 | 6 |
|---|---|---|---|
| 9 | 2700 | 360 | 54 |

2700
360
54
3114

Leading on to:

```
   346
x    9
  54 (9 x 6)
 360 (9 x 40)
2700 (9 x 300)
3114
 11
```

Then short multiplication:

```
   346
x    9
3114
 45
```

**TU x TU**
(Long multiplication – multiplication by more than a single digit)

72 x 38

Children will approximate first
72 x 38 is approximately 70 x 40 = 2800

| X | 70 | 2 |
|---|---|---|
| 30 | 2100 | 60 |
| 8 | 560 | 16 |

2100
560
60
16
2736

Leading on to 4 calculations by partitioning;

```
   72
x  38
  16 (8 x 2)
 560 (8 x 70)
  60 (30 x 2)
```

2100 (30 x 70)
27 36
1

*Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.*

## YEAR 6

**ThHTU x U**
(Short multiplication – multiplication by a single digit)

**HTU x TU**
(Long multiplication – multiplication by more than a single digit)

372 x 24

Children will approximate first
372 x 24 is approximately 400 x 25 = 10000

| X | 300 | 70 | 2 |
|---|---|---|---|
| 20 | 6000 | 1400 | 40 |
| 4 | 1200 | 280 | 8 |

6000
1400
1200
280
40
8
8928

372
x 24
1488
2
7440
1
8928
1

*Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.*

## PROGRESSION THROUGH CALCULATIONS FOR DIVISION

**Doubling and halving**
Knowing that halving is dividing by 2

**Deriving and recalling division facts**

***Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.***

| | |
|---|---|
| Year 2 | 2 times table |
| | 5 times table |
| | 10 times table |
| Year 3 | 2 times table |
| | 3 times table |
| | 4 times table |
| | 5 times table |
| | 8 times table |
| | 10 times table |
| Year 4 | Derive and recall division facts for all tables up to 12 x 12 |
| Year 5 & 6 | Derive and recall quickly division facts for all tables up to 12 x 12 |

**Using and applying division facts**
Children should be able to utilise their tables knowledge to derive other facts.
e.g. If I know 3 x 7 = 21, what else do I know?
30 x 7 = 210, 300 x 7 = 2100, 21 ÷ 7 = 3, 210 ÷ 30 = 7 etc.

**Dividing by 10 or 100**
Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.
Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

**Use of factors**

378 ÷ 21 378 ÷ 3 = 126 378 ÷ 21 = 18
126 ÷ 7 = 18

**Use related facts**

Given that 1.4 x 1.1 = 1.54
What is 1.54 ÷ 1.4, or 1.54 ÷ 1.1?

## RECEPTION and YEAR 1

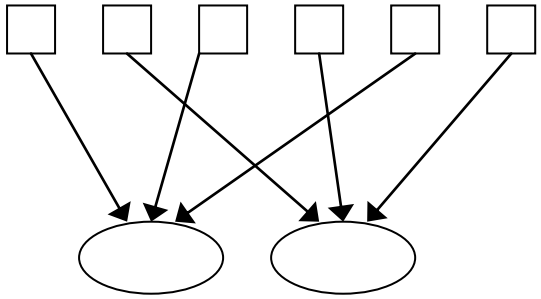
Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

## YEAR 2

Children will develop their understanding of division and use jottings and manipulatives to support calculation.

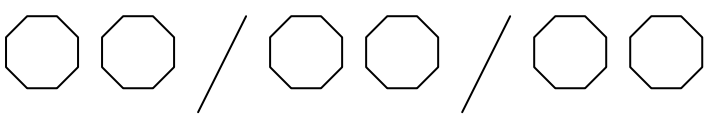
- ✓ **Sharing equally**

6 sweets shared between 2 people, how many do they each get?



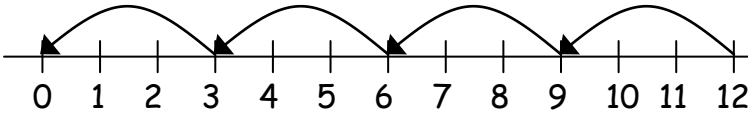
- ✓ **Grouping or repeated subtraction**

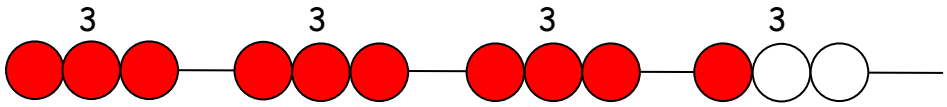
There are 6 sweets, how many people can have 2 sweets each?



✓ **Repeated subtraction using a number line or bead bar**

12 ÷ 3 = 4




The bead bar will help children with interpreting division calculations such as 10 ÷ 5 as 'how many 5s make 10?'

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

□ ÷ 2 = 4 20 ÷ △ = 4 □ ÷ △ = 4

## YEAR 3

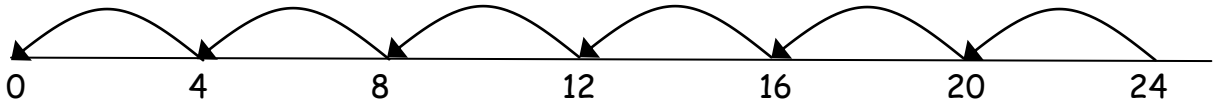
Ensure that the emphasis in Y3 is on grouping rather than sharing.

Children will continue to use:

✓ **Repeated subtraction using a number line**

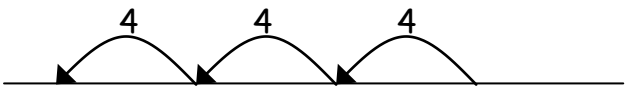
Children will use an empty number line to support their calculation.

24 ÷ 4 = 6




Children should also move onto calculations involving remainders and begin to round up or down depending on the context.

13 ÷ 4 = 3 r 1



0 1 5 9 13

- ✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

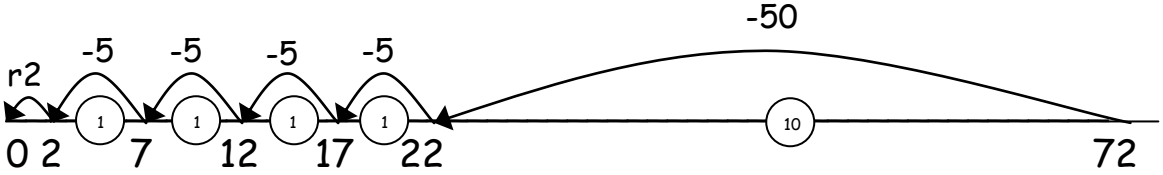
26 ÷ 2 = □ 24 ÷ △ = 12 □ ÷ 10 = 8

Children should realise that division is the opposite of multiplying and use times table facts to derive sums.

## YEAR 4

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

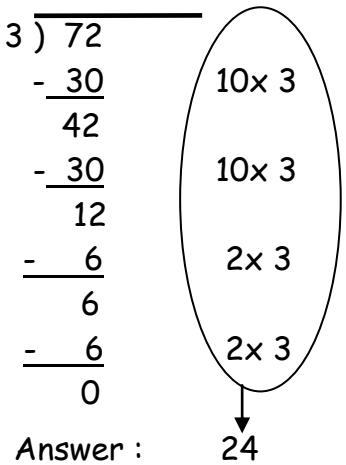
Take chunks away on a number line:




Then onto the vertical method *using bus stop at top for children to recognise symbol later on*:

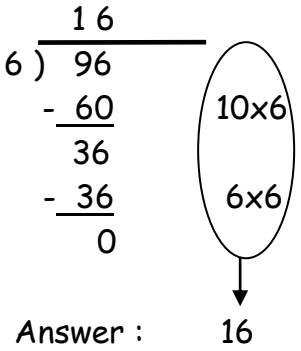
**Short division TU ÷ U**

72 ÷ 3




Leading to subtraction of other multiples.

96 ÷ 6




Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.
Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.
e.g. I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
Answer: 8 (the remaining 6 apples still need to be placed into a box)

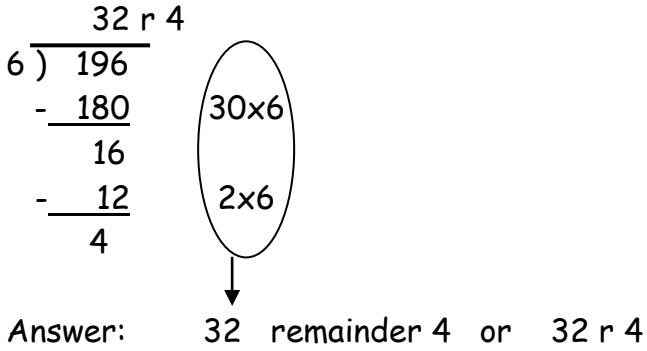
## YEAR 5

Children will continue to use written methods to solve short division HTU ÷ TU and HTU ÷ U.
e.g. 341 ÷ 14 or 167 ÷ 7

Children can start to subtract larger multiples of the divisor, e.g. 30x

**Chunking division HTU ÷ U**

196 ÷ 6




Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division.

For example 240 ÷ 52 is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

As children develop confidence in times tables, the written method of 'short division' (bus stop) can be introduced, HTU ÷ U (if they know their corresponding facts).

e.g. 972 ÷ 6

```
   1  6  2
6 ) 9 3 7 2
```

## YEAR 6

Children will continue to use written methods to solve short division TU ÷ U and HTU ÷ U.

**Long division HTU ÷ TU**

972 ÷ 36

```
    27
36 ) 972
   - 72
    252
   - 252
      0
```

Using the knowledge that: 10 x 36 = 360
5 x 36 = 180
2 x 26 = 72

Answer: 27

Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3\ ^{2}/_{10}$ which could then be written as $3\ ^{1}/_{5}$ in it's lowest terms.

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

87.5 ÷ 7

```
   1 2. 5
7 ) 8 17. 35
```

Answer : 12.5

Children need to have an understanding of when they need to leave a remainder, decimal, fraction or to round up or down for the final answer.